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STRUCTURE OF A COMPRESSION SHOCK IN TWO-PHASE MEDIA

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Shockwaves originating in supersonic two-phase flows can be considered to consist of two zones — the compression shock that is realized at the shock velocity greater than the frozen speed of sound [1, 2], and the relaxation zone. Although the structure of the relaxation zone has been investigated in sufficient detail [1-5], the structure of the compression shock has not.

It is generally understood that a lifting medium is described by Hugoniot relationships during passage through a shock, while particles "ignore" the compression shock. Meanwhile, experimental data [3] display a sufficiently strong influence of the compression shock on heterogeneous inclusions if the particle size does not exceed 20-25 μm.

For sufficiently intensive shocks, shocks originate in the domain with as large as a two-phase medium flows around a solid boundary, where the stream parameters change substantially within distances commensurate with the dimensions of the inclusions. Models of a continuous medium do not hold in these zones and the structure of such flows can be investigated only within the framework of kinetic theory [6]. On the other hand, for a weak shock intensity the thickness of the compression shock can exceed the dimensions of the inclusions by an order and more, which permits utilization of the approximation of a continuous medium to investigate the compression shock structure.

Here we consider the structure of a compression shock by using the kinetic and hydrodynamic descriptions of a two-phase medium. It is shown that the presence of particles results in an increase in the compression shock thickness, where the particles exert the greatest influence on the density and velocity profiles. It is found that taking account of the dissipative components on both the kinetic and the hydrodynamic level broadens the limits of applicability of these approaches somewhat.

1. Since a compression shock especially influences fine particles, for simplicity in our analysis we shall henceforth limit ourselves to a study of small size inclusions (not more than 10 μm, for instance), which permits utilization of the diffusion approximation in individual cases.

We consider a two-phase medium as a dynamic system of interacting particles. We write the kinetic equations for the particles of each phase (subscripts i, j) in the form

$$Df_i/Dt = Q(f_i, f_j). \tag{1.1}$$

Here Q is the interaction integral, and f(t, **x**, **v**) is the particle distribution function of one phase.

Two interaction scales, short-range and sliding collisions, can be separated for an analysis of Q in application to a two-phase mixture [7]. For the short-range collision, direct contact occurs between two (or more) particles with an exchange of mass, momentum, and energy.

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During sliding collisions there is no interparticle contact, but an exchange of momentum and energy is effected by means of the particles of the other phase. On the hydrodynamic level, this is analogous to the interaction of two or more closely located particles through a lifting medium [8] for heterogeneous inclusions. Representing the integral as the sum

$$Q = Q^* + \tilde{Q},$$

where Q^* is the contact interaction integral and $\tilde{Q}$ is the component taking account of the sliding collisions, and considering the effect of intermediate collisions as a continuous sequence of small random changes in the velocity and coordinates, we obtain [7]

$$\tilde{Q} = \sum_{i,\gamma=1}^{3} \left(\frac{\partial}{\partial v_i} \pi_d^p \frac{\partial f}{\partial v_\gamma} - \frac{\partial}{\partial v_i} \pi_r^p f + \frac{\partial}{\partial x_i} \pi_d^q \frac{\partial f}{\partial x_\gamma} - \frac{\partial}{\partial x_i} \pi_r^q f \right).$$

Here v is the velocity, and π_j^i are the coefficients of friction r and diffusion d in the phase space (p, q). A similar expression for Q can be obtained if the source in the Langevin equation is considered a function of not only the velocity but also of the coordinates [9], which denotes taking account of dissipative processes at the microlevel. The coefficients π_j^i have the form (the derivation of these coefficients is the same as for the Fokker–Planck equations [10], and is not presented here)

$$\pi_d^p = M_1 \int f \Delta p^2 d\mathbf{y}, \quad \pi_d^q = M_2 \int f \Delta q^2 d\mathbf{y}, \tag{1.2}$$
$$\pi_r^p = M_3 \int f \Delta p d\mathbf{y}, \quad \pi_r^q = M_4 \int f \Delta q d\mathbf{y}, \quad \mathbf{y} = \{x, v\},$$

where Δp, Δq are the characteristic magnitudes of the momentum and coordinates, and M_i is the interaction transformant.

If the kinetic equation can be written as

$$Df/Dt = Q/\varepsilon$$

(ε is a small quantity), then according to the method of Struminskii [11], when using the Maxwell distribution in a zero approximation, we find

$$\pi_q^p = \Delta p^2 \Delta q \varkappa^* \varphi_p^r, \quad \pi_d^p = \Delta q^3 \varkappa^* \varphi_p^d, \quad \pi_r^q = \Delta q^2 \varkappa^* \varphi_q^r,$$
$$\pi_d^q = \Delta p \Delta q \varkappa^* \varphi_q^d, \quad \varkappa^* = 3{,}2\,(2kT/m),$$

where I is the Boltzmann constant, m is the particle mass, T is the temperature, and ϕ_j^i is the consistency coefficient determined from experimental data or additional hypotheses.

We write the integral Q^* in the form [12]

$$Q^* = \sum^{N} \int (W_- ff - W_+ ff)\, d\mathbf{y} d\mathbf{y},$$
$$W_\mp = \sum_\alpha \int M_\alpha |\mathbf{v} \cdot \mathbf{n}| v dv,$$

where n is the normal N is the total number of particles, $\sum_\alpha$ denotes summation over all the arriving (or departing) particles at the time of contact interaction.

If the interaction transformant M_α is considered for specular interaction of pairs of particles, then Q^* degenerates into the Boltzmann integral [13].

2. To investigate the structure of a compression shock we initially use the Mott-Smith approach [14] according to which

$$f = a_- f_- + a_+ f_+, \quad a_- + a_+ = 1. \tag{2.1}$$

Here a_- and a_+ are weight coefficients, f_- and f_+ values of the distribution function before and after the shock. Taking the square of the velocity v^2 as the trial function in the generalized transport equation (which is obtained by traditional means and is not presented here), we obtain in conformity with the relations (2.1)

$$\frac{da_-}{dx} = -\beta a_- (1 - a_-) - 2\pi_r^p \rho_- v_- - 6\pi_d^p (a_+ \rho_+ + a_- \rho_-), \tag{2.2}$$
$$\beta = \alpha [2\rho_- v_- R(T_- - T_+)]^{-1},$$

$$\alpha = \int (v^2 - v'^2) f_+(v) f_-(v) B(\theta, V)\, d\theta d\varepsilon dv_* dv,$$
$$B(\theta, V) = s(\theta, V) V \sin\theta,$$

where s is the differential scattering section, V is the thermal velocity, the subscript * is mute, and ρ is the density. Assumptions are made in deriving (2.2) which are used later: the flow of the two-phase medium is stationary and one-dimensional, the interaction processes are Markovian in nature, the interaction transformants correspond to a specular collision, there are no phase transformations, the coefficients π_j^i are calculated under the assumption that the distribution functions in (1.2) are Maxwellian, and the two-phase medium is considered in the diffusion approximation.

The last two assumptions are made to simplify the analysis. The solution of (2.2) has the form

$$a_- = \frac{l + \beta^* + (\beta^* - l)\exp(2l\beta x^*)}{1 + \exp(2l\beta x^*)},$$
$$l^2 = \frac{\eta}{\beta} + \left(\frac{\beta + \xi}{2\beta}\right)^2, \quad a_+ = 1 - a_-, \quad \xi = 6\pi_d^p(\rho_- - \rho_+), \tag{2.3}$$
$$\eta = 6\pi_d^p\rho_+, \quad \beta^* = \frac{\beta + \xi}{2\beta}, \quad x^* = x + \text{const.}$$

The coordinate x^* is introduced for convenience in the selection of the origin of the coordinate system. Integral relationships on the shock, whose derivation is presented below, are used in solving (2.2). Taking account of the solution (2.3) and the integral relationships on the shock, the equations for the running values of the parameters on the compression shock are as follows:

$$\rho(x) = \rho_-[\rho_+/\rho_- + a_-(1 - \rho_+/\rho_-)], \quad v(x) = \rho_- v_- / \rho(x),$$
$$e(x) = e_- + v_-^2/2 - v^2(x)/2.$$

The coefficient ϕ_p^d is determined from (2.3) and the condition s for the velocity in the compression shock by using experimental data [3]. It is later assumed that all the coefficients are constant and equal to ϕ_p^d.

Taken as the momentum scale Δp is the sound velocity in a heterogeneous mixture

$$\Delta p = m\sqrt{dp^*/d\rho},$$

multiplied by the particle mass m, and the particle diameter of heterogeneous inclusions as the characteristic dimension Δq.

The Liepmann method [15] is used within the framework of the kinetic theory to study the shock structure in the second approach, whereupon (1) is reduced to the form

$$v\frac{df}{dx} = \nu(f_0 - f) + \frac{d}{dx}\pi_d^q\frac{df}{dx} + \frac{d}{dx}\pi_r^q f,$$
$$\nu = -\pi_1 \int \sum_\alpha \int\int r dr d\varphi M_\alpha f dy, \quad \pi_1 = N r_0^3 / v_\Phi, \tag{2.4}$$
$$f_0 = -\frac{\pi_1}{\nu}\int \sum_\alpha \int\int r dr d\varphi M_\alpha f f dy$$

with the boundary conditions $f(x = \pm\infty) = 0$. Without examining different particular cases of the analytic solution of (2.4), which are presented in [16], it is solved numerically with iterations in f_0 and ν, where the Maxwell distribution is again used for the function f in a zero approximation.

Since the appropriate moments and integral relations for the shock must be used in the Mott-Smith method, it is necessary to represent the results in terms of hydrodynamic variables (including also for the Liepmann method). We note that the majority of experimental data on the shock structure is presented in precisely hydrodynamic variables.

The characteristics of a two-phase flow in a compression shock, obtained by the Mott-Smith and Liepmann methods, respectively, for different external stream parameters are represented in Figs. 1 and 2; all the quantities are referred to the parameters ahead of the shock. Polystyrene particles of 2 μm diameter were selected as heterogeneous inclusions. Given for comparison in Fig. 1 are the curves 6-8 in v, T, ρ for pure air for $M_- = 10$, 1, 2, 4 in ρ, T, v for $M_- = 10$ for a two-phase flow; 3, 5 in T, v for $M_- = 5$. The mean free path was determined from the relationship

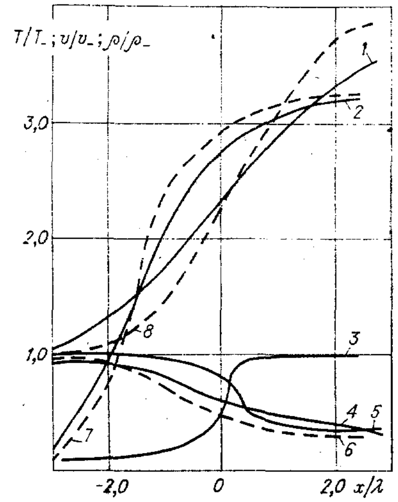

Fig. 1

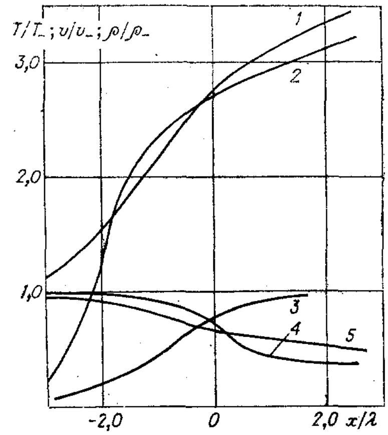

Fig. 2

$$\lambda = (\mu_-/\rho_-)(\pi/(2RT_-))^{1/2}.$$

It follows from Fig. 1 that the presence of particles results in weak asymmetry in the density and velocity profiles in the shock upon application of the Mott-Smith method, and on the other hand, diminishes the asymmetry in the density profiles when using the Liepmann method (Fig. 2; see Fig. 1 for explanation). In both approaches the heterogeneous phase results in an increase in shock thickness, which is computed from the dependence

$$L = \frac{\rho_+ - \rho_-}{(d\rho/dx)_{\max}}.$$

If a criterion is introduced that characterizes the degree of particle influence

$$\Gamma^* = \left|\frac{\Gamma_0/\Gamma_n - \Gamma/\Gamma_-}{\Gamma_0/\Gamma_n}\right|, \tag{2.5}$$

where Γ is the hydrodynamic parameter whose profile is considered, Γ_0 is the value for the lifting medium without inclusions, and Γ_n is the value for the homogeneous medium before the shock, then it is seen that the particles exert the greatest influence on the velocity and density. The change in the temperature profile because of the presence of particles is insignificant: 1-5% for both approaches for M_- = 1.01-10.

3. It is seen from an analysis of Figs. 1 and 2 that within the framework of the diffusion approximation with small free stream Mach numbers the extent of the compression shock considerably exceeds the mean free path. The shock is still not blurred here. Consequently, it is expedient to consider the shock structure in the hydrodynamic formulation also. Therefore we express the stationary mass, momentum, and energy conservation equations in one-dimensional formulations, which are obtained as the appropriate moments of the kinetic equation (1.1) (the normalization condition is $\int f du = \rho$):

$$\frac{d}{dx}\rho u = \Lambda^*(1) + \Lambda^W(1) + \Lambda\Omega(1) + g_{l\gamma}\pi_d^q \frac{d^2\rho}{dx^2} - g_l\pi_r^q \frac{d\rho}{dx}; \tag{3.1}$$

$$\frac{d}{dx}\left(\rho u^2 + P_p - \frac{4}{3}\mu\frac{du}{dx}\right) = \Lambda^*(u) + \Lambda^W(u) + \Lambda\Omega(u) + \tag{3.2}$$

$$+\Big[-\tau_{l\gamma}\pi_d^p \frac{d}{dx}\rho\frac{du}{dx} - \tau_{l\gamma}\rho\pi_d^p \frac{d^2u}{dx^2} - \rho\tau_l\pi_r^p \frac{du}{dx} + g_{l\gamma}\pi_d^q \frac{d^2\rho u}{dx^2} -$$

$$- g_{l\gamma}\pi_d^q \frac{d}{dx}\rho\frac{du}{dx} - g_{l\gamma}\pi_d^q \frac{d}{dx}\rho\frac{du}{dx} + g_{l\gamma}\rho\pi_d^q \frac{d^2u}{dx^2} - g_l\pi_r^q \frac{d}{dx}\rho u + g_l\rho\pi_r^q \frac{du}{dx}\Big];$$

$$\frac{d}{dx}\left[(e + u^2/2)\rho u - \lambda_0\frac{dT}{dx} - \frac{4}{3}\mu u\frac{du}{dx}\right] = \Lambda^*(E) + \Lambda^W(E) + \tag{3.3}$$

$$+\Lambda\Omega(E) + \Big(-\tau_{l\gamma}\pi_d^p \frac{d}{dx}\rho\frac{dE}{dx} - \rho\tau_{l\gamma}\pi_d^p \frac{d^2E}{dx^2} - \rho\tau_l\pi_r^p \frac{dE}{dx} + g_{l\gamma}\pi_d^q \frac{d^2E\rho}{dx^2} -$$

$$- g_{l\gamma}\pi_d^q \frac{d}{dx}\rho\frac{dE}{dx} - g_{l\gamma}\pi_d^q \frac{d}{dx}\rho\frac{dE}{dx} + \rho g_{l\gamma}\pi_d^q \frac{d^2E}{dx^2} - g_l\pi_r^q \frac{d\rho E}{dx} +$$

$$+ g_l\rho\pi_r^q \frac{dE}{dx}\Big), \quad \rho = \varphi_g\rho_g + \varphi_p\rho_p, \quad E = e + u^2/2,$$

$$e = \varphi_g e_g + \varphi_p e_p, \quad \lambda_0 = \varphi_g \lambda_g + \varphi_p \lambda_p, \quad \rho u = \varphi_g (\rho u)_g + \varphi_p (\rho u)_p,$$
$$\tau_{l\gamma} = \tau_l \tau_\gamma, \quad g_{l\gamma} = g_l g_\gamma, \quad \Lambda^* (\psi) = \int \psi Q^* du,$$
$$\Lambda^W (\psi) = \int \psi Q_W du, \quad \Lambda\Omega (\psi) = \int \psi Q\Omega du.$$

Here τ_i, g_i are the process characteristic time and length, μ is the viscosity, and P_p is the pressure. After conversion of (3.1)-(3.3), we have the first integrals

$$\rho (u + g_l \pi_r^q) - g_{l\gamma} \pi_d^q \frac{d\rho}{dx} = \rho_- u_- + \Phi^m; \tag{3.4}$$

$$\rho u (u + g_l \pi_r^q) + \left(\tau_{l\gamma} \pi_d^p \rho + g_{l\gamma} \pi_d^q \rho - \frac{4}{3} \mu\right) \frac{du}{dx} - g_{l\gamma} \pi_d^q u \frac{d\rho}{dx} + P_p + J_u = \rho_- u_-^2 + P_{p-} + \Phi^u; \tag{3.5}$$

$$\begin{gathered} \rho u E - \lambda_0 \frac{dT}{dx} - \frac{4}{3} \mu u \frac{du}{dx} + \tau_{l\gamma} \pi_d^p \rho \frac{dE}{dx} - g_{l\gamma} \pi_d^q \frac{dE\rho}{dx} + \\ + 2 g_{l\gamma} \pi_d^q \rho \frac{dE}{dx} + g_l \pi_r^q \rho E + J_E = \rho_- u_- E_- + g_l \pi_r^q \rho_- E_- + \Phi^E, \end{gathered} \tag{3.6}$$

where

$$J_u = \int \left(\rho \tau_{l\gamma} \pi_d^p \frac{d^2 u}{dx^2} + \rho \tau_l \pi_r^p \frac{du}{dx} - \rho \pi_d^q g_{l\gamma} \frac{d^2 u}{dx^2} - g_l \rho \pi_r^q \frac{du}{dx}\right) dx;$$
$$J_E = \int \left[(\rho \tau_{l\gamma} \pi_d^p - \rho g_{l\gamma} \pi_d^q) \frac{d^2 E}{dx^2} + (\rho \tau_l \pi_r^p - g_l \rho \pi_r^q) \frac{dE}{dx} \right] dx;$$
$$\Phi^\psi = \int \left[\sum_{i=1}^{3} (\Lambda_i^\psi - \Lambda_{i0}^\psi) \right] dx; \quad \Lambda_0 = \Lambda_-;$$
$$\sum_{i=1}^{3} \Lambda_i^\psi = \Lambda^* (\psi) + \Lambda^W (\psi) + \Lambda\Omega (\psi).$$

From (3.3)-(3.5) we obtain integral relationships for the compression shock

$$\rho u = \rho_- u_- + \Phi^m; \tag{3.7}$$

$$\rho u^2 + P_p = \rho_- u_-^2 + P_{p-} + \Phi^u; \tag{3.8}$$

$$\rho u E = \rho_- u_- E_- + \Phi^E. \tag{3.9}$$

To close the system (3.1)-(3.9) we write the equation of state within the framework of the diffusion approximation [1]:

$$P_p = \rho R_p T, \; R_p = R/(1 + \varkappa), \; \varkappa = \rho_p/\rho_g. \tag{3.10}$$

4. The solution of (3.4) for c_p = const and taking account of the dependences (3.6)-(3.8), (3.10) is the following

$$\begin{gathered} -\frac{c_1}{2c_2} \ln |c_2 u^2 + c_3 u + c_4| + \left(\frac{c_1 c_3}{2c_2} - c_0\right) \frac{1}{\sqrt{c_3^2 - 4c_2 c_4}} \times \\ \times \ln \left| \frac{2c_2 u + c_3 - \sqrt{c_3^2 - 4c_2 c_4}}{2c_2 u + c_3 + \sqrt{c_3^2 - 4c_2 c_4}} \right| = x + \text{const}, \\ c_0 = g_{l\gamma} \pi_d^q (\Phi^m + \rho_- u_-), \quad c_1 = \mu^*, \\ c_2 = (\rho_- u_- + \Phi^m)(1 - R_p/2c_p), \quad c_4 = \frac{R_p}{c_p} (\rho_- u_- E_- + \Phi^E), \\ c_3 = g_l \pi_r^q (\rho_- u_- + \Phi^m) - g_{l\gamma} \pi_d^q \frac{d\Phi^m}{dx} + J_u - (\rho_- u_-^2 + \Phi^u + P_{p-}), \\ \mu^* = \tau_{l\gamma} \pi_d^p \rho + g_{l\gamma} \pi_d^q \rho - \frac{4}{3} \mu. \end{gathered} \tag{4.1}$$

Another solution for

$$4c_2 c_4 > c_3^2$$

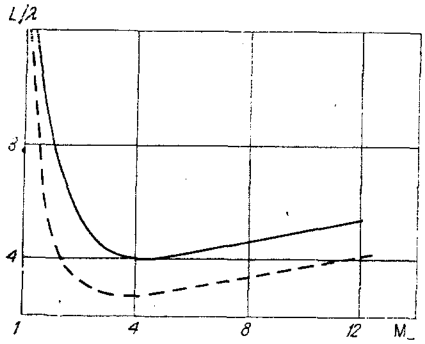

Fig. 3

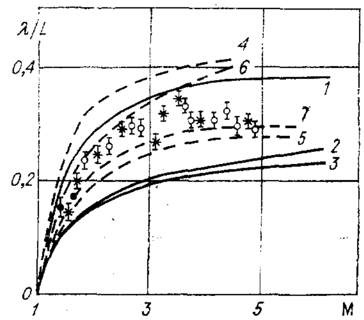

Fig. 4

has no physical meaning. The solution of (4.1) is realized by iterations in the integral J_u and conditions ahead of the shock and the Maxwell distribution were selected in a zero approximation.

A particle-Maxwell sphere model, i.e.,

$$\beta = \left[\frac{90\,M_-}{3+M_-^2}\,\frac{(M_-^2-1)^2}{16\,M_-^4-(3+M_-^2)^2}\sqrt{\frac{2\pi}{15}}\right]\lambda^{-1},$$

was taken in all the computations presented, as were also the characteristic quantities on the micro- and macrolevels, equal to

$$\tau_l = \Delta qm/\Delta p,\ g_l = \Delta q.$$

Figure 3 shows the change in the relative shock thickness as a function of M_- for clean and dusty air (dashed and solid lines, respectively). Again polystyrene with a 2 μm particle diameter and 0.01 volume concentration was chosen as particles. As for the kinetic description, the presence of particles results in an increase in compression shock thickness. If (2.5) is considered for this approach, we see that the particles exert the greatest influence on the velocity and density, as in the kinetic approach.

Figure 4 shows a comparison of the results obtained when utilizing the kinetic and hydrodynamic approaches, and experimental data [17, 18] are presented for a pure gas. Lines 1-3 are the results of the hydrodynamic approach, the Mott-Smith method and the Liepmann method, respectively, for a flux with 2-μm-diameter polystyrene particles (0.01 volume concentration). Given for comparison are the dependences of the dimensionless transit length (λ/L) on M_- for pure air without taking account of the dissipative terms (4 is the hydrodynamic approach and 5 is the Mott-Smith model). Curves 6 and 7 are data for pure air with dissipative terms taken into account (the hydrodynamic approach and the Mott-Smith model). It can be noted that within the framework of the approach used above the domain of application of the models is increased somewhat: the range of the hydrodynamic description of the shock structure grows to $M_- \approx 2.5$, while the kinetic models describe a shock to $M_- \approx 2$ more accurately. This closure of the application of the models can be explained by taking account of the dissipative components in the kinetic equation, which results, in turn, in the appearance of additional diffusion and convective transfer terms in the mass, momentum, and energy conservation equations.

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DYNAMICS OF DROPLET BREAKUP IN SHOCK WAVES

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Study of the principles of acceleration and fragmentation of droplets during interaction with a high speed gas flow is of interest because of its important practical applications (for example, atomization of liquids in various technological processes, energy generation equipment, detonation wave propagation in gas—droplet systems, etc.). In particular, the problem of heterogeneous detonation in a gas—droplet system requires detailed study of the processes of acceleration, deformation, fragmentation, ignition, and combustion of droplets within shock waves at Mach numbers M = 2-6 for Weber numbers $We = \rho u^2 d_0 \sigma^{-1} > 10^3$ and Reynolds numbers $Re = \rho u d_0 \mu^{-1} > 10^3$. Here ρ, u, μ are the density, velocity and viscosity of the gas, d_0 is the initial droplet diameter, and σ is the liquid surface tension.

Numerous studies of droplet interaction with shock waves are reflected in reviews [1-3], which considered characteristic regimes of droplet breakup and indicated corresponding parameter ranges. Thus, according to [2], for $We > 10^3$, $Re > 10^3$, corresponding to the explosive droplet decay range, the following pattern exists. Over a time interval $0 < t < t_0$ (where $t_0 = d_0 \rho_\ell^{0.5} (\rho u^2)^{-0.5}$, ρ_ℓ being the liquid density) a droplet collapses into a disk of size $d \sim 3d_0$. At time $t \sim (0.1\text{-}0.5)t_0$ a thin layer of liquid begins to break away from the equatorial region of the deformed drop and then breaks into pieces. The dimensions of the microparticles thus formed are in the range $d \sim 1\text{-}10$ µm [4, 5]. Due to instability of the phase separation boundary at $t \simeq t_0$ explosive decay of the disk begins, reaching its greatest velocity at $t \sim (1.5\text{-}2)t_0$ and ending at $t \sim (4\text{-}5)t_0$. The dimensions of the particles formed by this explosive decay are of the order of the thickness of the disk into which the droplet was deformed at the time of maximum deformation $d \sim (0.1\text{-}0.2)d_0$ [1]. The nucleus of the disintegrating drop moves along a trajectory $x d_0^{-1} \sim (0.5\text{-}1.4)t^2 t_0^{-2}$ [6]. However, despite the large number of experiments which have been performed, many questions concerning droplet breakup in shock waves remain little studied. Among these, in particular, are the effect of viscosity on droplet destruction dynamics, the size of the microparticles formed, the process of evaporation of the disintegrating droplet, etc.

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